CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9231 FURTHER MATHEMATICS

9231/21 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a “fortuitous” answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Find T by equating dv/dt at $t = T$ to 6:	$4T - 4 = 6$, $T = 2.5$	M1 A1		
	Find radial component v^2/r of acceln. at $t = T$:	$v^2/r = (2T^2 - 4T + 3)^2 / 0.25$			
	(M0 if T not given a value)	$= (11/2)^2 \times 4 = 121$ [m s^{-2}]	M1 A1	4	**4**
	SR: Max M1 (1/4) if linear and angular confused				
2 (i)	Find ω^2 from SHM eqn. $d^2x/dt^2 = -\omega^2 x$ at C:	$0.625 = 10\omega^2$, $\omega^2 = 0.0625$ *or* 1/16	B1		
	Find period T [s] from $T = 2\pi/\omega$: (ft on ω^2)	$T = 2\pi/¼ = 8\pi$ (not 25.1)	B1√	2	
(ii)	Find amplitude a [m] from $v_C^2 = \omega^2(a^2 - 10^2)$:	$6^2 = \omega^2(a^2 - 10^2)$			
		$a^2 = 6^2 \times 16 + 10^2$, $a = \sqrt{676} = 26$	M1 A1	2	
	Find time from C to M, e.g.:	$\omega^{-1}\sin^{-1}(10/a) + \omega^{-1}\sin^{-1}½$ *or*			
		$\omega^{-1}\cos^{-1}(-10/a) - \omega^{-1}\cos^{-1}½$ *or*			
		$½T - \omega^{-1}\cos^{-1}(10/a) - \omega^{-1}\cos^{-1}½$	M1		
	(AEF throughout)	$= \omega^{-1}\{0.3948 + \pi/6\ [= 0.5236]\}$ *or*			
		$\omega^{-1}\{1.9656 - \pi/3\ [= 1.0472]\}$ *or*			
		$\omega^{-1}\{\pi - 1.760 - \pi/3\ [= 1.0472]\}$	A1		
		$= 1.579 + 2.094$ *or* $7.862 - 4.189$			
		or $12.567 - 4.704 - 4.189$			
		or 4×0.9184; $= 3.67$ [s]	A1; A1	4	**8**
3	Find v^2 from conservation of energy:	$½mv^2 = ½mu^2 + mga(1 - \cos\theta)$	M1 A1		
	Find R by using $F = ma$ radially:	$R = mg\cos\theta - mv^2/a$	B1		
	Eliminate v^2 to find R: AG	$R = mg(3\cos\theta - 2) - mu^2/a$	M1 A1	5	
	Find u^2 or v^2 in terms of $\cos\theta$ when $R = 0$:	$u^2 = ag(3\cos\theta - 2)$ *or*			
		$v^2 = ag\cos\theta$	B1		
	EITHER: Replace $\cos\theta$ in energy eqn with $v = 2u$:	$4u^2 = u^2 + 2ag - ⅔(u^2 + 2ag)$			
		or $u^2 + 2ag - 8u^2$	M1 A1		
	OR: Find $\cos\theta$ and substitute in energy eqn:	$[v^2/ag =]\ 4(3\cos\theta - 2) = \cos\theta$			
		$\cos\theta = 8/11$			
		$4u^2 = u^2 + 2ag(1 - 8/11)$	(M1 A1)		
	Hence find u:	$u = \sqrt{(2ag/11)}$ *or* $0.426\sqrt{(ag)}$	A1	4	**9**
4 (i)	Take moments for rod about B:	$W \times a\cos 30° + 3W \times 2a\cos 30°$			
	(or with $\cos 30° = \sqrt{3}/2$)	$= T \times 2a\cos 30°$	M1 A1		
	Hence find tension T:	$T = 7W/2$	A1		
	(Can earn M1 A0 A1 if e.g. sin 30° wrongly used)				
	Find modulus λ using Hooke's Law:	$T = \lambda(2a - 3a/5) / (3a/5)$			
		$\lambda = (3/7)(7W/2) = 3W/2$	M1 A1	5	

Question Number	Mark Scheme Details			Part Mark	Total
(ii)	*EITHER:* Find horizontal component of force F at B:	$X = T\cos 30°$ $= (7\sqrt{3}/4)\,W$ *or* $3.03\,W$	B1√		
	Find vertical component: (X, Y ft on T)	$Y = 4W - T\sin 30° = 9W/4$	B1√		
	Find magnitude of F:	$F = \sqrt{(X^2 + Y^2)}$ $= (\sqrt{57}/2)\,W$ *or* $3.77[5]\,W$	B1		
	Find direction of F (AEF): (A0 if direction unclear)	Upward force at angle to AB of $\tan^{-1} Y/X = \tan^{-1} 3\sqrt{3}/7$ $= 36.6°$ *or* 0.639 radians	M1 A1		
	OR: Find component along CB:	$F_1 = (4W + T)\sin 30° = 15W/4$	(B1√)		
	Find normal component: (F_1, F_2 ft on T)	$F_2 = (4W - T)\cos 30° = (\sqrt{3}/4)W$	(B1√)		
	Find magnitude of F:	$F = (\sqrt{57}/2)\,W$ *or* $3{\cdot}77[5]\,W$	(B1)		
	Find direction of F (AEF): (A0 if direction unclear)	Upward force at angle to CB of $\tan^{-1} F_2/F_1 = \tan^{-1} \sqrt{3}/15$ $= 6.6°$ *or* 0.115 radians	(M1 A1)		
	OR: Find component parallel to string CA:	$\pm F_1 = T - 4W \sin 30° = 3W/2$	(B1√)		
	Find normal component: (F_1, F_2 ft on T)	$\pm F_2 = 4W \cos 30° = 2\sqrt{3}\,W$	(B1√)		
	Find magnitude of F:	$F = (\sqrt{57}/2)\,W$ *or* $3.77[5]\,W$	(B1)		
	Find direction of F (AEF): (A0 if direction unclear)	Upward force at angle to AC of $\tan^{-1} F_2/F_1 = \tan^{-1} 4/\sqrt{3}$ $= 66.6°$ *or* 1.16 radians	(M1 A1)	5	**10**
5	For A & B use conservation of momentum, e.g.: (m may be omitted here and below)	$3mv_A + 2mv_B = 3mu$	M1		
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Combine to find v_B :	$v_B = 3(1 + e)\,u/5$	A1		
	For B & C use conservation of momentum, e.g.:	$2mv_B' + mv_C = 2mv_B$	M1		
	Use Newton's law of restitution (consistent signs):	$v_C - v_B' = e'\,v_B$	M1		
	Combine to find v_C and v_B':	$v_C = 2(1 + e')\,v_B/3$ $= 2(1 + e)(1 + e')\,u/5$ AG	A1		
		$v_B' = (2 - e')\,v_B/3$ $= (1 + e)(2 - e')\,u/5$	A1	7	
	Find ratios or values of v_A, v_B', v_C from momentum:	$3v_A = 2v_B' = v_C$ $[= u]$	B1		
	Find e from first collision eqns, e.g.: (or find e' and then use $3v_A = 2v_B'$)	$v_A = (3 - 2e)\,u/5 = u/3$ *or* $v_B = ½\,(3u - u)$ *or* $(⅓ + e)\,u$ $= 3(1 + e)\,u/5$, $e = ⅔$	M1 A1		
	Find e' from second collision eqns, e.g.:	$2v_B' = v_C$ so $2(2 - e') = 2(1 + e')$ *or* $v_C = 2(1 + ⅔)(1 + e')\,u/5 = u$ *or* $v_B' = (1 + ⅔)(2 - e')\,u/5 = u/2$ $e' = ½$	M1 A1	5	**12**
6	Equate pooled estimate of σ^2 to 12:	$(11 - 5^2/N + 160 - 10^2/10)\,/$ $(N + 10 - 2) = 12$	M1 A1		
	Formulate and solve relevant quadratic eqn. for N:	$12N^2 - 65N + 25 = 0$, $N = 5$	M1 A1	4	**4**

Question Number	Mark Scheme Details			Part Mark	Total
7	Find Σx via sample mean $\overline{x}$:	$\Sigma x = 8\overline{x} = 8 \times ½ (1.17 + 2.03)$ $= 8 \times 1.6 = 12.8$	M1 A1		
	Find estimate of population variance s^2:	$t \sqrt{(s^2/8)} = ½ (2.03 – 1.17)$ [= 0.43]	M1		
	Use of correct tabular value (1·96 leads to 23·2):	$t_{7, 0.975} = 2.36[5]$	A1		
	(to 3 d.p.)	$s^2 = 0.2645$ *or* 32/121 *or* 0.5143^2	A1		
	Find Σx^2 from s^2:	$s^2 = \{\Sigma x^2 - (\Sigma x)^2/8\}/7$			
	(M0 for $s^2 = \{…\}/8$)	$\Sigma x^2 = 7 \times 0.2645 + 12.8^2/8 = 22.3$	M1 A1	7	**7**
8 (a) (i)	Find correlation coefficient r from $r^2 = b_1 b_2$:	$r = \sqrt{(0.38 \times 0.96)} = 0.604$	M1 *A1	2	
(ii)	State both hypotheses (B0 for r …):	H_0: $\rho = 0$, H_1: $\rho > 0$	B1		
	State or use correct tabular one-tail r-value:	$r_{10, 5\%} = 0.549$	*B1		
	State or imply valid method for reaching conclusion:	Reject H_0 if $\lvert r \rvert$ > tab. value (AEF)	M1		
	Correct conclusion (AEF, dep *A1, *B1):	There is positive correlation	A1	4	
(b)	State or use relevant tabular two-tail r-value:	$r_{16, 5\%} = 0.497$ (*or* $r_{15, 5\%} = 0.514$)	M1		
	Find least possible value of n:	$n_{min} = 16$	A1	2	
	SR M1 A1 for stating 16 without explanation B1 for stating 15 without explanation B1 for finding or stating one-tail result 12				**8**
9 (i)	Relate $P(X > x)$ to number of flaws (AEF):	$P(X > x)$ = P(zero flaws in x m)	B1		
	Relate this to Poisson distn. (AEF):	$= P_0(0.8x) = e^{-0.8x}$ **A.G.**	B1	2	
(ii)	Find P(number of flaws ≥ 1): (M0 if "1 –" omitted)	$1 – P_0(0.8 \times 4) = 1 – e^{-3.2}$ $= 1 – 0.0408 = 0.959$	M1 A1	2	
(iii) (a)	Find or state distribution function $F(x)$:	$F(x) = P(X \le x) = 1 – P(X > x)$ $= 1 – e^{-0.8x}$	B1	1	
(b)	Find or state probability density function $f(x)$: **S.R.** Deduct A1 if **(a)**, **(b)** interchanged	$f(x) = dF/dx = 0.8 e^{-0.8x}$	M1 A1	2	
(c)	Formulate equation for either quartile value Q :	$F(Q) = 1 – e^{-0.8Q} = ¼$ *or* ¾	M1		
	Find lower quartile Q_1 : (AEF)	$Q_1 = 1.2 \ln 4/3$ [= 0.360]	A1		
	Find upper quartile Q_3 : (AEF)	$Q_3 = 1.2 \ln 4$ [= 1.733]	A1		
	Find interquartile range (allow $Q_1 – Q_3$):	$Q_3 – Q_1$ [= 1.2 ln 3] = 1.37	A1	4	**11**

Question Number	Mark Scheme Details			Part Mark	Total
10	Calculate gradient b_1 in $y - \overline{y} = b_1(x - \overline{x})$:	$S_{xy} = 313.28 - 50.8 \times 56.9/10$ $= 24.228$ $S_{xx} = 284.16 - 50.8^2/10 = 26.096$			
	(PA –1 so max 4/5 for 0.93, giving $y = 7.48$)	$b_1 = S_{xy} / S_{xx} = 0.928$	M1 A1		
	Find y when $x = 7$ from regression line of y on x:	$y = 56.9/10 + b_1 (7 - 50.8/10)$ $= 5.69 + 0.928 (7 - 5.08)$ $[= 0.928 \times 7 + 0.976]$	M1 A1		
		$y = 7.47$ (allow 7.48 or 7.5)	A1	5	
	SR If regression line of x on y used:	$S_{yy} = 347.59 - 56.9^2/10 = 23.829$			
		$b_2 = S_{xy} / S_{yy} = 1.017$	(M1)		
		$7 = 50.8/10 + b_2 (y - 56.9/10)$ $= 1.017\, y - 0.707$	(M1 A1)		
	(can earn at most 4/5)	$y = 7.58$ (allow 7.6)	(A1)		
	Find differences (e.g. $y - x$) and sample mean:	1 0.7 1.3 0.2 0.1 0.9 0.8 0.5 0.1 0.5 $\overline{d} = 6{\cdot}1 / 10 = 0.61$	M1 A1		
	Estimate population variance (to 3 s.f.): (allow biased here: 0.1469 *or* 0.3833^2)	$s^2 = (5.19 - 6.1^2/10) / 9$ $= 0.1632$ *or* 0.404^2	B1		
	State hypotheses (AEF; B0 for $\overline{x}$), e.g.:	H_0: $\mu_y - \mu_x = 0.4$, H_1: $\mu_y - \mu_x > 0.4$	B1		
	Calculate value of t:	$t = (\overline{d} - 0.4)/(s/\sqrt{10}) = 1.64$	M1 A1		
	State or use correct tabular t-value: (or can compare $\overline{d}$ with 0.634)	$t_{9,\,0.95} = 1.83[3]$	B1		
	Consistent conclusion (AEF, ft on both t-values):	[Accept H_0:] No improvement of more than 0.4	B1√		
	Wrong test can earn only B1 for hypotheses and B1 for conclusion			8	**13**

Question Number	Mark Scheme Details			Part Mark	Total
11A	Find MI of disc about O:	$I_{\text{disc}} = ½\, 2ma^2 = ma^2$	B1		
	Find MI of ring about O:	$I_{\text{ring}} = 2m\,(3a)^2 = 18\,ma^2$	B1		
	EITHER: Find MI of any rod about O:	$I_{rod} = ⅓\,(3m/2)a^2 + (3m/2)(2a)^2$			
		$= (13/2)\,ma^2$	B1		
	OR: Find MI of 2 collinear rods about O:	$⅓\,(9m/2)(3a)^2 - ⅓\,(3m/2)a^2$			
		$= 13\,ma^2$	(B1)		
	Find MI of object about O:	$I_O = ma^2 + 18\,ma^2 + 4(13/2)\,ma^2$			
		$= 45\,ma^2$ AG	B1	4	
	Find MI of object about axis at O // to tangent:	$I_O' = ½ I_O$	M1		
	Find MI of object about tangential axis at A:	$I_A = I_O' + 10m\,(3a)^2$			
		$= (225/2)\,ma^2$	M1 A1	3	
	Find new MI when particle is attached at C:	$I_A' = I_A + 3m\,(6a)^2$			
		$= (441/2)\,ma^2$	M1		
	Find and use initial angular speed:	$\omega_0 = u/3a$	B1		
	Find gain in P.E. at instantaneous rest:	$(10mg \times 3a + 3mg \times 6a)(1 - \sin\theta)$	M1 A1		
		$= 45mga/2 + 27mga/2$			
		or $48mga\,(1 - \sin\theta) = 36mga$	A1		
	Find u by equating to rotational KE:	$= ½\, I_A'\, \omega_0^{\,2}$	M1		
		$u^2 = (36/441) \times 36ag$ [$441=21^2$]			
		$u = (12/7)\,\surd(ag)$ *or* $1.71\,\surd(ag)$	A1	7	**14**
	SR: Taking AC at $\sin^{-1}(¼)$ to vertical:	P.E. $= 48mga\,(1 - \cos \sin^{-1}(¼))$			
		$= 1.524\,mga$	(A0)		
	(max 6/7)	$u = 0.35[3]\,\surd(ag)$	(A1)		
	SR: Overlooking added particle can earn M0 B1 M1 A0 A0 M1 A0 (max 3/7)				
11B	State suitable distribution:	Geometric	B1	1	
	State (at least) null hypothesis: (AEF) (B0 for "It is a good fit")	H_0: Distn. fits data *or* $p = 0.6$	B1		
	Find exp. values using $200pq^{x-1}$ with $p = 0.6$, $q = 0.4$:	120 48 19.2 7.68			
	(ignore incorrect final value here,	3.072 1.2288 0.8192	M1		
	e.g. 0.4915 which can earn max 5/8				
	Combine last 3 cells since exp. value < 5:	O: . . . 6			
		E: . . . 5.12	B1		
	Calculate χ^2 (result correct to 3 s.f.):	$\chi^2 = 0.3 + 0.5208 + 0.4083$			
		$+ 2.8519 + 0.15125 = 4.23[2]$	M1 A1		
	State or use consistent tabular value (to 3 s.f.):	5 cells: $\chi_{4,0.95}^{\ 2} = 9.488$			
	[or if 2 or no cells combined:	6 cells: $\chi_{5,0.95}^{\ 2} = 11.07$			
		7 cells: $\chi_{6,0.95}^{\ 2} = 12.59$			
	or if 4 cells combined, as with 0·4915:	4 cells: $\chi_{3,0.95}^{\ 2} = 7.815$]	B1√		
	Valid method for reaching conclusion:	Accept H_0 if χ^2 < tabular value	M1		
	Conclusion (AEF, requires both values correct): (Allow A1 for "It is a good fit")	$4.23 < 9.49$ so distn fits *or* $p = 0.6$	A1	8	
	Find prob. p of at least one 6 on 5 throws of one die:	$p = 1 - 0.75^5 = 0.7627$ (4 s.f.)	M1 A1		
	Find prob. of at least one 6 on exactly 4 of 10 dice:	${}^{10}C_4 \times$; $p^4\,(1 - p)^6$	M1; M1		
		$= 210 \times 6.043 \times 10^{-5}$			
		$= 0.0126$ *or* 0.0127	A1	5	**14**